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NEW WINGS APPROACH – WINGS OF FINITE SUM OF INFLUENCES†

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Abstract

The Weighted Influence Non-linear Gauge System (WINGS) is a multi-criteria decision making method. It is used as a structural model to analyze the interaction between the elements. WINGS is a relatively new method, but it is increasingly used. Some authors have already improved the original method and also included fuzzy numbers in it. The convergence problem may occur, when the total strength-influence matrix is derived. We proposed that instead of using an infinite sum of terms in the total strength-influence matrix a finite sum of influences is used. The aim of this study is to propose a new concept of WINGS method - finite sum of WINGS influences (FSI WINGS). The FSI WINGS method gives comparable results to the WINGS method and is therefore suitable for use.

Key words: *multi-criteria decison making method, finite sum of WINGS influences, fuzzy numbers, convergence problem*

1. INTRODUCTION

Multi criteria decision making (MCDM) is the collective term for mathematical methods used to find solutions to decision problems with multiple (usually) conflicting goals (Eggers et al., 2019). There are many different methods that deal with MCDM, such as Analytic Hierarchical Process (AHP), Technique for Order Performance by Similarity to Ideal Solutions (TOPSIS), VIse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR), Decision Making Trial and Evaluation Laboratory (DEMATEL) and others (Tavana et al., 2021). One of them is the Weighted Influence Non-linear Gauge System (WINGS), which is similar to the DEMATEL method.
WINGS method was proposed by Michnik (Michnik, 2013). It is used to evaluate elements of the system, where not all elements are equally important (do not have the same strength). So, the WINGS improve DEMATEL method, because it includes strength of elements, while the DEMATEL method does not. In WINGS method elements are treated as a system of relationships, focusing on the internal importance of the system's element (strength) and its interrelationship with other elements (influence). The method is combined with graph theory to analyze the logical relationship between elements and formulate a direct strength-influence matrix *D*. In this process the information derived from the total strength-influence matrix *T* is used to calculate the exerted effects (impact) and the received effects (receptivity) of each element, as well as the involvement (engagement) and role (position) of the elements in the system to quantify the interrelationships between each element and the importance of each element in the system (Wang et al., 2021).

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WINGS method has been used in the various fields such as economy (Banaś & Michnik, 2019), construction management (Fedorczak-Cisak et al., 2020), business processes (Kashi & Franek, 2014), urbanization (Adamus-Matuszyńska et al., 2019) and others.
The acronym of the WINGS method reflects important features of the method (Michnik, 2013):

- Weighted – means that the method considers the internal strength (importance of the element);
- Influence – stresses the crucial role of interrelationships between elements;
- Non-linear - the mathematical processing of input data introduces the non-linearity into the model;
- Gauge System is self-explanatory.

Nowadays, many decisions are made in an uncertain environment. Zadeh (1965) introduced fuzzy set theory as a mathematical tool to represent and manage vagueness and uncertainty in decision making (Tseng, 2009). Therefore, many authors use triangular fuzzy numbers (TFNs) in MCDM to represent decision makers evaluations. The WINGS method considers the strength and influence of elements. When the TFNs are included, uncertainty can be added to the method.
In WINGS method, when the total strength-influence matrix *T* is calculated, the convergence problem may occur. If all row sums or column sums of the normalized strength-influence matrix *X* are equal to 1, then the matrix is stochastic (Bronson, 1989) and the infinite sum of normalized influences does not converge, therefore the total strength-influence matrix *T* does not exist (Chen et al., 2017). Therefore, some authors have proposed a different normalization, to solve convergence problem (Michnik, 2013).
The aim of this paper is to introduce a new concept in the WINGS method - Finite sum of WINGS influences method (FSI WINGS), to solve the convergence problem.
The rest of this paper is organized as follows. In Section 2, we introduce the WINGS method, triangular fuzzy numbers, the fuzzy WINGS method and a new approach – Finite sum of WINGS influences method (FSI WINGS). In Section 3, we show an example with different types of WINGS methods (WINGS method, fuzzy WINGS method and FSI WINGS method. At the end of the paper, we give a summary of our research.

2. METHODS

2.1. WINGS

The procedure for implementing the WINGS method is presented below through the following steps:

1. step: Generate the strength-influence matrix *D*
Experts select the $n>2$ elements that constitute the system and then determine the relationships between them. It is recommended to create a map of influences that represents the system. The nodes on the graph represent the elements of the system and the arrows represent the nonzero influence of one element on another. When the system is determined, the decision makers assess all the influences of one element on another and strength of each element using scalar values, where 0 represents no influence/strength and 4 represents very high influence/strength. The assessments of the decision makers, influences and strengths, are inserted into the direct strength influence matrix *D*. This is a $n \times n$ matrix with elements d_{ij}. Values representing strength of elements are inserted into principal diagonal d_{ii} (strength of element i). Values representing influences of elements

are inserted in such a way that for $i \neq j$, d_{ij} is influence of element i on element j (Michnik, 2013).

2. step: Derive the normalized strength-influence matrix X
There are different ways of normalization of the strength-influence matrix D, that effect on the final results.
$X = \left[x_{ij} \right]$ is the normalized strength-influence matrix and x_{ij} is calculated as follows

$$x_{ij} = \frac{d_{ij}}{s}, \; j = 1, ..., n; \tag{1}$$

Where the normalizing factor s is given by:

$$s = \max_{1 \leq j \leq n} \sum_{j=1}^{n} d_{ij} \tag{2}$$

$$s = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \tag{3}$$

$$s = \max(\max_{1 \leq i \leq n} \sum_{j=1}^{n} d_{ij}, \max_{1 \leq j \leq n} \sum_{i=1}^{n} d_{ij}) \tag{4}$$

3. step: Derive the total strength-influence matrix T
Total strength-influence matrix T is an infinitive sum of influences among factors. $T = \left[t_{ij} \right]$ is calculated as follows

$$T = X + X^2 + X^3 + ... + X^n + ... = X(I - X)^{-1}. \tag{5}$$

4. step: Sum the rows and columns and construct the causal diagram
For each element in the system the rows sum R_i (total impact of elements) and the column sum C_j (total receptivity of elements) of the matrix T is calculated as follows:

$$R_i = \sum_{j=1}^{n} t_{ij}, \; i = 1, ..., n, \tag{6}$$

$$C_j = \sum_{i=1}^{n} t_{ij}, \; j = 1, ..., n. \tag{7}$$

Then $R_i + C_i$ and $R_i - C_i$ are calculated. $R_i + C_i$ is called prominence value and represent the importance of element *i*. $R_i - C_i$ called relations value. The causal diagram consists of $R_i + C_i$ on the horizontal axis, while $R_i - C_i$ on the vertical axis divides the elements into cause (positive values) and effect (negative values) groups.

2.2. Fuzzy sets

Fuzzy sets were first developed by Zadeh (1965). In decision making, fuzzy sets are used to represent and deal with uncertainty. In fuzzy sets each number between 0 and 1 indicates a partial truth, so we can express and handle uncertain judgments mathematically (Wu, 2012). Fuzzy set is defined as follows:

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) \middle| x \in X \right\}, \mu_A(x) : \mathrm{X} \rightarrow [0,1], \tag{8}$$

where $\mu_{\tilde{A}}$ represents membership function. Membership functions can have different shapes (triangular, trapezoidal or Gaussian) (Li, Li, Liu, & Deng, 2018). In fuzzy WINGS triangular fuzzy numbers (TFNs) are commonly used (Tavana et al., 2021). They can be used to express human linguistic evaluations. TFN is defined as $\tilde{A} = (l, m, u)$ where l is the lower bound, m is the middle value and u is the upper bound of TFN, and with the membership function (Aouag, Soltani, & Mouss, 2020)

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < l \text{ or } x > u \\ (x-l)/(m-l), & l \le x \le m. \\ (u-x)/(u-m) & m < x \le u \end{cases} \tag{9}$$

2.3. Fuzzy WINGS

Fuzzy WINGS method consists of the same steps as the WINGS method, except that fuzzy WINGS method uses TFN evaluations, so some steps are adapted to the use of fuzzy numbers. In first step the decision maker assess all the influences of one element on another and strength of each element using linguistic terms. In third step, where the total strength-influence matrix $\tilde{T}$ is generated, the lower bounds, the middle values, and the upper bounds, are derived separately.

Step 1: Generate the strength-influence matrix $\tilde{D}$
Step one is similar to the WINGS method, except that in fuzzy WINGS method the linguistic assessments of decision makers are converted into the corresponding TFNs (Table **1**).

Table 1. Linguistic terms, their corresponding scalar numbers and their corresponding triangular fuzzy numbers

Corresponding scalar value	**Linguistic terms**	**Abbreviation**	**Corresponding triangular fuzzy number**
0	No influence/strength	NI/NS	(0, 0, 0.25)
1	Very low influence/strength	VLI/VLS	(0, 0.25, 0.5)
2	Low influence/strength	LI/LS	(0.25, 0.5, 0.75)
3	High influence/strength	HI/HS	(0.5, 0.75, 1)
4	Very high influence/strength	VHI/VHS	(0.75, 1, 1)

Source: (Tavana et al., 2021)

All values, influences and strengths of the elements are then written into the strength-influence matrix $\tilde{D}$.

$$\tilde{D} = \begin{pmatrix} (l_{11}, m_{11}, u_{11}) & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & (l_{22}, m_{22}, u_{22}) & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & (l_{nn}, m_{nn}, u_{nn}) \end{pmatrix} \tag{10}$$

Step 2: Derive the normalized strength-influence matrix $\widetilde{X}$

The strength-influence matrix $\widetilde{D}$ normalized using normalization (11), to obtain the normalized strength-influence matrix $\widetilde{X}$.

$$x_{ij} = \frac{(l_{ij}, m_{ij}, u_{ij})}{\sum_{j=1}^{n} u_{ij}}, i, j = 1,...,n \tag{11}$$

Step 3: Derive the total strength-influence matrix $\widetilde{T}$

Total strength-influence matrix $\widetilde{T} = (T_l, T_m, T_u)$ is derived separately for the lower bounds (12), the middle values (13), and the upper bounds (14).

$$T_l = X_l (I - X_l)^{-1} \tag{12}$$

$$T_m = X_m (I - X_m)^{-1} \tag{13}$$

$$T_u = X_u (I - X_u)^{-1} \tag{14}$$

For the defuzzification the following equation is chosen.

$$T = \frac{T_l + 4T_m + T_u}{6} \tag{15}$$

Step 4: Sum the rows (6) and the columns (7) and then construct the causal diagram.

2.4. Convergence problem in WINGS

When the total strength-influence matrix T is derived, the convergence problem may occur, because the infinite sum of terms does not converge:

Let $X = \left[x_{ij} \right]_{n \times n}$, $0 \le x_{ij} < 1, 0 < \sum_{j=1}^{n} x_{ij} \le 1,\ 0 < \sum_{i=1}^{n} x_{ij} \le 1$.

Then $\sum_{i=1}^{h} X^i = X + X^2 + X^3 \ldots + X^h = X(I + X + X^2 + X^3 + \ldots + X^{h-1}) \left[(I - X)(I - X)^{-1} \right] =$
$= X(I - X^h)(I - X)^{-1}$.

The total strength-influence matrix T can be obtained by $T = X + X^2 + X^3 + ... = \lim_{h \to \infty} \sum_{i=1}^{h} X^i$, when $\lim_{h \to \infty} X^h = [0]_{n \times n}$. If all row sums are not equal to 1, then we can provide $\lim_{h \to \infty} X^h = [0]_{n \times n}$ (Chen et al., 2017). If all row sums are equal to 1, then $\lim_{h \to \infty} X^h \ne [0]_{n \times n}$, and a convergence problem arises.

2.4.1. Example with convergence problem

There is one example, where the convergence problem arises in WINGS method. Figure 1 shows the graph of system with corresponding linguistic evaluations.

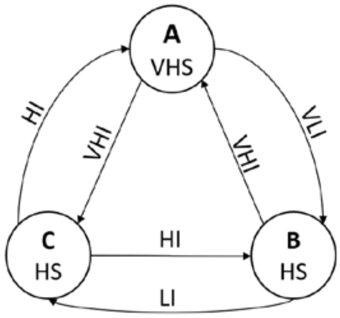

Figure 1. The graph of system (Example 1)

Assessments of decision maker are written into the strength-influence matrix D.

$$D = \begin{pmatrix} 4 & 1 & 4 \\ 4 & 3 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

Table 2. The normalized strength-influence matrix X with Row sums

X	A	B	C	Row sums
A	0,444	0,111	0,444	1
B	0,444	0,333	0,222	1
C	0,333	0,333	0,333	1

The normalized strength-influence matrix X has all row sums equal to 1, therefore the total strength-influence matrix T cannot be determined, because $T = X + X^2 + X^3 + ... + X^n + ... = X(I - X)^{-1}$ does not exist. The WINGS method cannot be used.
Some authors solved convergence problem with different normalization, where instead of (2) take (3) (Michnik, 2013).
We propose a new approach – Finite sum of WINGS influences (FSI WINGS), that interpretaion folowing.

2.4.2. Interpretation of new approach

The total strength-influence matrix T represents the sum of all normalized influences and strength of elements. $T = X + X^2 + X^3 + ...,$ where X represents direct normalized influences and strength, and $X^2, X^3, X^4, ...$ represent indirect influences and strength of the element. The different levels of influences and strengths are determined. The first level are

direct influences and strength collected in matrix X. The second level are indirect influences and strength through one element, represented in matrix X^2. The third level are indirect influences and strength through two elements gathered in X^3 and so on. The indirect influences and strength of elements on a selected element are initially large and then decrease with the levels. The higher levels of influences are smaller and contribute less to the total influence.
So, our new proposal differs from the WINGS method in step 3, where the total strength-influence matrix T is calculated. Instead of an infinite sum, a finite sum (a certain number of terms) is used (one term less than the number of elements).

2.5. Finite sum of WINGS influences – FSI WINGS

Step 1: Generate the strength-influence matrix D

Step 2: Derive the normalized strength-influence matrix X (1)

Step 3: Derive the total strength-influence matrix T

This step differs in the FSI WINGS method from the WINGS method. In FSI WINGS, matrix T is a finite sum of influences between factors. T is calculated as follows

$$T = X + X^2 + ... + X^{n-1}. \quad (16)$$

Step 4: Sum the rows (6) and the columns (7) and then construct the causal diagram

3. EXAMPLE

An example from the literature (Tavana et al., 2021), where the fuzzy WINGS is examined, is selected. The data is computed with different types of WINGS method (WINGS, fuzzy WINGS and FSI WINGS). Then, the results are compared with each other.

Figure 2 shows the map of influence relationships between factors, with the corresponding linguistic evaluations of decision maker.

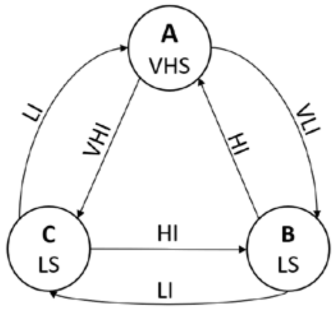

Figure 2. The graph of system (Example 2)

3.1. WINGS method

First the calculation with WINGS method is made. Assessments of decision maker are written into the strength-influence matrix D.

$$D = \begin{pmatrix} 4 & 1 & 4 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{pmatrix}$$

The normalized strength-influence matrix X derived by (1), but instead of maximum of row sums (2) we take sum of all row sums (3).

$$X = \begin{pmatrix} 0.174 & 0.043 & 0.174 \\ 0.130 & 0.087 & 0.087 \\ 0.087 & 0.130 & 0.087 \end{pmatrix}$$

In Table 2 the results derived with the WINGS method are presented.

Table 3. T matrix obtained with WINGS method, Row sums (R), Column sums (C), Prominence values (R_i+C_i) and Relations values (R_i-C_i)

T (WINGS)	A	B	C	R	C	$R_i + C_i$	$R_i - C_i$
A	0.252	0.095	0.247	0.594	0.591	1.185	0.003
B	0.193	0.125	0.144	0.462	0.390	0.851	0.072
C	0.147	0.170	0.139	0.456	0.531	0.986	-0.075

3.2. Fuzzy WINGS method

The strength-influence matrix D shows assessments written in corresponding TFNs.

$$D = \begin{pmatrix} (0.75,\ 1,\ 1) & (0,\ 0.25,\ 0.5) & (0.75,\ 1,\ 1) \\ (0.5,\ 0.75,\ 1) & (0.25,\ 0.5,\ 0.75) & (0.25,\ 0.5,\ 0.75) \\ (0.25,\ 0.5,\ 0.75) & (0.5,\ 0.75,\ 1) & (0.25,\ 0.5,\ 0.75) \end{pmatrix}$$

The normalized strength-influence matrix X of upper bounds, middle values and lower bound derived by (12), (13) and (14).

$$X = \begin{pmatrix} (0.100,\ 0.133,\ 0.133) & (0.000,\ 0.033,\ 0.067) & (0.100, 0.133,\ 0.133) \\ (0.067,\ 0.100,\ 0.133) & (0.033,\ 0.067,\ 0.100) & (0.033,\ 0.067,\ 0.100) \\ (0.033,\ 0.067,\ 0.100) & (0.067,\ 0.100,\ 0.133) & (0.033,\ 0.067,\ 0.100) \end{pmatrix}$$

In Table 3 the results derived with the fuzzy WINGS method are presented.

Table 4. T matrix obtained with fuzzy WINGS method, Row sums (R), Column sums (C), Prominence values (R_i+C_i) and Relations values (R_i-C_i)

T (fuzzy WINGS)	A	B	C	R	C	$R_i + C_i$	$R_i - C_i$
A	0.168	0.061	0.166	0.394	0.401	0.796	-0.007
B	0.134	0.089	0.098	0.321	0.273	0.593	0.048
C	0.100	0.123	0.096	0.318	0.359	0.677	-0.041

3.3. FSI WINGS method

The normalized strength-influence matrix X derived by (1).

$$X = \begin{pmatrix} 0.174 & 0.043 & 0.174 \\ 0.130 & 0.087 & 0.087 \\ 0.087 & 0.130 & 0.087 \end{pmatrix}$$

In Table 4 the results derived with the FSI WINGS method are presented (16).

Table 5. T matrix obtained with FSI WINGS method, Row sums (R), Column sums (C), Prominence values (R_i+C_i) and Relations values (R_i-C_i)

T (FSI WINGS)	A	B	C	R	C	$R_i + C_i$	$R_i - C_i$
A	0.225	0.078	0.223	0.526	0.524	1.049	0.002
B	0.172	0.112	0.125	0.408	0.346	0.754	0.062
C	0.127	0.157	0.121	0.405	0.469	0.873	-0.064

3.4. Summary of example

Figure 2 shows the results derived with different types of WINGS method (WINGS method, fuzzy WINGS method and FSI WINGS method) in cause and effect diagram.

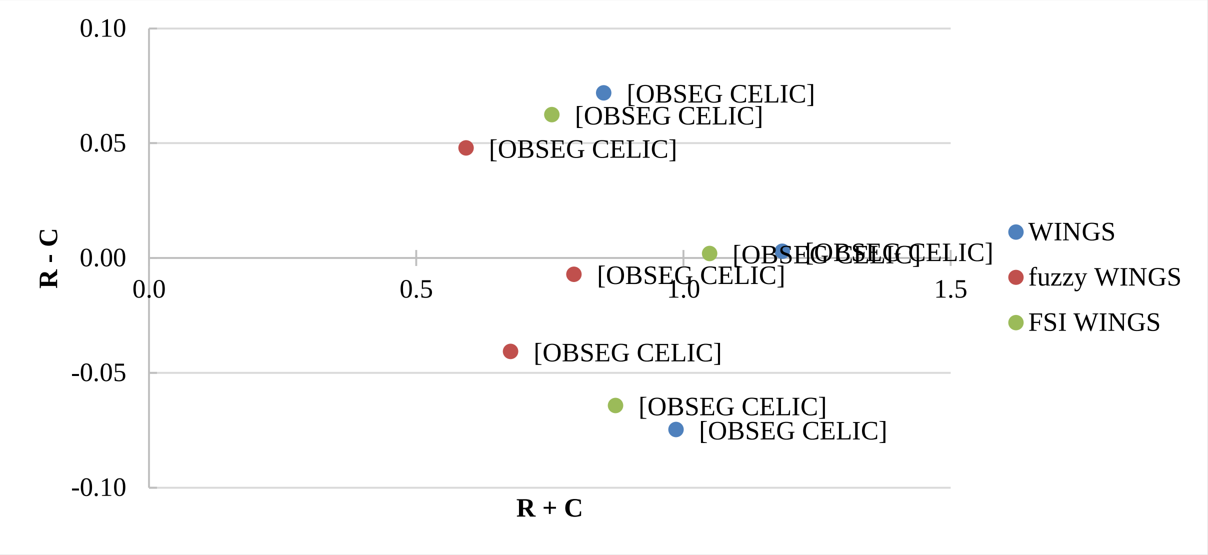

Figure 3. Cause and effect factor calculated with different types of WINGS method

The distribution and relationships between the elements of the all three methods are similar as can be seen in the diagram. Elements in the cause group and elements in the effect group are the same for all methods. We can see, that the absolute values of the elements, that depend on the normalization of assessments, are different, while the relative values of the elements, that represent the relationships between the elements, remain similar regardless of which method is used. We can conclude that the FSI WINGS method is suitable since it provides comparable results to the WINGS method and fuzzy WINGS method.

4. CONCLUSION

In this paper, we have discussed the WINGS method and fuzzy WINGS method. WINGS method, as a descendant of DEMATEL method, inherits all merits, good and bad, of its predecessor. One of drawback is that the convergence problem occurs when the infinite sum of normalized influences does not converge.

We proposed the new method Finite sum of WINGS influences – FSI WINGS. Instead of an infinite sum, the FSI WINGS method uses a finite sum (a certain number of terms) of influences. This method has successfully solved the problem of convergence. In order to validate the new method, it was compared with the WINGS method and the fuzzy WINGS method from the literature. The results show, that FSI WINGS method is suitable for the application, since it gives similar results to the other two methods.

In summary, the FSI WINGS method can be used as an MCDM method to evaluate the relationships between elements in system in many different fields.

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NOVI WINGS PRISTUP – WINGS KONAČNI ZBIR UTICAJA

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Izvod

WINGS (engl. Weighted Influence Non-linear Gauge System) je višekriterijmska metoda odlučivanja. Koristi se kao strukturni model za analizu interakcije između elemenata. WINGS je relativno nova metoda, ali sve više koristi. Neki autori su već unapredili originalnu metodu i uključili u nju fazi brojeve. Problem konvergencije može nastati kada se izvodi matrica konačnog zbira uticaja. Predložili smo da se umesto korišćenja beskonačnog zbira članova u matrici ukupne jačine uticaja koristi konačan zbir uticaja. Cilj ovog rada je da se predloži novi concept WINGS metode – WINGS konačni zbir uticaja (engl. Finite Sum of WINGS Influences - FSI WINGS). FSI WINGS metoda daje rezultate koji su uporedivi sa WINGS metodom, pa je stoga pogodan za upotrebu.

Ključne reči: *metoda višekriterijumskog odlučivanja, WINGS konačni zbir uticaja, fazi brojevi, problem konvergencije*

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